Indian Statistical Institute, Bangalore
B. Math (II)
Second Semester 2017-18
Semester Examination : Statistics (II)

Date: 09-05-2018 **Maximum Score 40** **Duration: 3 Hours**

1. Let $X_1,\ X_2, \cdots, X_n$ be a random sample from $N(\mu, \sigma^2)$; $\mu \in I\!R$ and $\sigma^2 > 0$,both unknown. Obtain *Cramer Rao Lower Bound (CRLB)* for the variance of an unbiased estimator for σ^2. When can you obtain an unbiased estimator for σ^2 whose variance attains *CRLB?* Substantiate.

   $[6 + 4 = 10]$

2. To study the effect of examination, researchers measured anxiety scores, once on a regular day and once on an examination day, for the same random sample of 11 students. The data collected are shown below. Do the data support, at $\alpha = 0.05$, the hypothesis that examination increases the anxiety levels? State clearly the assumptions you make. Find p-value. Find 90% *upper confidence bound* (UCB) for the mean increase in anxiety scores.

Sr. No. $\longrightarrow$ Score$\downarrow$	1	2	3	4	5	6	7	8	9	10	11
Regular Day	16	03	17	03	19	15	24	23	03	12	32
Exam Day	32	22	23	13	20	29	11	25	13	20	30

   $[10 + 2 + 4 = 16]$

3. Oxide layers on semiconductor wafers are etched in a mixture of gases to achieve the proper thickness. The variability in the thickness of these oxide layers is a critical characteristic of the wafer, and a low variability is desirable for subsequent processing steps. Two different mixtures of gases are being studied to determine whether one is superior in reducing the variability of the oxide thickness. Let X and Y respectively denote the oxide layer thickness for the two samples. The following data were collected $\sum_{i=1}^{12}\left(X_i - \overline{X}\right)^2 = 57.06$ and $\sum_{i=1}^{16}\left(Y_i - \overline{Y}\right)^2 = 68.1$. List carefully the assumptions you must make before formulating the problem. Formulate and carry out the problem of testing of hypotheses to check whether there is any evidence to indicate that either gas is preferable at $\alpha = 0.05$? Find $p - value$.

   $[2 + 8 + 2 = 12]$

4. Let $X_1, X_2, \cdots, X_n$ be a random sample from $f(x|\theta)$, $\theta \in \Theta \subset \mathbb{R}$. Let $L(\theta|\boldsymbol{x}) = \prod_{i=1}^{n} f(x_i|\theta)$ be the likelihood function. Let $\widehat{\theta}$ denote the *mle* of θ. Consider the problem of testing $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$. Let $\lambda(\boldsymbol{x}) = \frac{L(\theta_0|\boldsymbol{x})}{\sup\limits_{\theta\in\Theta} L(\theta|\boldsymbol{x})}$ be the likelihood ratio. Let the regularity conditions $(A1)-(A6)$ listed in the class hold. Then prove that under H_0, $-2\log\lambda(\boldsymbol{X}) \xrightarrow{d} \chi_1^2$.

   $[12]$